Equilibrium Equations (2D)

\[ \sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_o = 0 \]

Average stresses

\[ \sigma_{avg} = \frac{N}{A} \quad \tau_{avg} = \frac{V}{A} \]

Allowable stresses

\[ \sigma_{allow} = \frac{\sigma_{fail}}{F.S.} \quad \tau_{allow} = \frac{\tau_{fail}}{F.S.} \]

\[ \sigma_{avg} \leq \sigma_{allow} \quad \tau_{avg} \leq \tau_{allow} \]

Normal strain

\[ \epsilon_{avg} = \lim_{\Delta s \to 0} \frac{\Delta s_i - \Delta s}{\Delta s} ; \quad L' = (1 + \epsilon)L_0 \]

Shear strain

\[ \gamma_{nt} = \frac{\pi}{2} \lim \theta' \]

Hooke’s Law (Elastic Region)

\[ \sigma = E\epsilon \quad \tau = G\gamma \quad G = \frac{E}{2(1 + \nu)} \]

Poisson’s ratio

\[ \nu = -\frac{\epsilon_{lat}}{\epsilon_{long}} \]
MAE 213 Final Exam Formula Sheet

Axial Normal Stress: \[ \sigma = \frac{N(x)}{A(x)} \]

Axial Elongation: \[ \delta = \int_{0}^{l} \frac{N(x)}{EA(x)} \, dx \quad \delta = \sum \frac{N_i L_i}{EA_i} \]

Thermal Displacement: \[ \delta_r = \alpha \Delta TL \]

Torsion - Shear Stress: \[ \tau = \frac{T \rho}{J} \]

Torsion: Polar Moments of Inertia of Circular Areas

Solid: \[ J = \frac{\pi}{2} c^4 \];
Tubular: \[ J = \frac{\pi}{2} (c_o^4 - c_i^4) \]

Torsion: Angle of Twist

Solid: \[ \phi = \int_{0}^{l} \frac{T(x)}{J(x)G} \, dx \];
Tubular: \[ \phi = \sum \frac{T_i L_i}{J_i G} \]

Power Transmission: \[ P = T \omega \]
MAE 213 Final Formula Sheet

Centroids:
\[
\bar{y} = \frac{\int y \, dA}{A}; \quad \bar{z} = \frac{\int z \, dA}{A}; \quad \text{or} \quad \bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}; \quad \bar{z} = \frac{\sum \bar{z}_i A_i}{\sum A_i};
\]

Moment of Inertia of Areas
\[
I_x = \int_A z^2 \, dA; \quad I_z = \int_A y^2 \, dA
\]

Parallel Axis Theorem
\[
I_x = I_{x'} + d^2 A; \quad I_z = I_{z'} + d^2 A
\]

Bending: Distributed Load, Shear Force, and Moment
\[
\frac{dV}{dx} = -w(x); \quad \frac{dM}{dx} = V(x)
\]

Deflection and Moment
\[
EI \frac{d^2 v}{dx^2} = M
\]

Bending: Flexure Formula
\[
\sigma_x = -\frac{M_z y}{I_z}
\]
Practice Problems

Problem 1
The solid circular shaft with original length 600 mm and radius 20 mm is subjected to the axial tensile force $P$. The bar is made of a material having a stress-strain diagram as shown.

A. If a force of 80kN is applied, what is the length of the bar after the force is removed?  
B. For A, what is the change in cross-sectional area? Poisson’s ratio is 0.33.  
C. If a force of 250 kN is applied, what is the length of the bar after the force is removed?

Problem 2
The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load. The Young’s modulus for steel is 200 GPa and the Young’s modulus for brass is 125 GPa.

Problem 3
The steel shaft is made from two segments: $AC$ has a diameter of 0.5 in, and $CB$ has a diameter of 1 in. If it is fixed at its ends $A$ and $B$ and subjected to a torque of 500lb-ft. Determine the maximum shear stress in the shaft. The shear modulus $G = 10.8 \times 10^3$ ksi.
Problem 4
The beam has the T-shaped cross-section as shown below. The External loading and dimensions are also shown.

A. Draw the shear and moment diagrams.
B. Calculate the centroid location and the moment of inertia.
C. Determine the maximum tensile bending stress in the beam and its locations.
D. Determine the maximum compressive bending stress in the beam and its locations.

Problem 5
Determine the equations of the elastic curve for the beam using the $x_1$ and $x_2$ coordinates. Specify the slope at $A$ and the maximum displacement of the shaft. EI is constant.
Problem 1

Solutions

\[ P = 80 \text{kN} \implies \sigma_{\text{applied}} = \frac{P}{A} = \frac{80 \text{kN}}{\pi (20 \text{mm})^2} = 63.67 \text{ MPa} \]

Since \( \sigma_{\text{applied}} \leq 150 \text{ MPa} \) only in elastic region.

\[ \sigma_{\text{applied}} = E \varepsilon \]

\[ \varepsilon = \frac{63.67 \text{ MPa}}{120 \text{ GPa}} = 0.000531 \]

\[ E = \frac{150 \text{ MPa}}{0.00125} = 120 \text{ GPa} \]

\[ \varepsilon = \frac{\Delta l}{l_0} \implies \lambda f = 600.32 \text{ mm} \]

\[ V = -\frac{\varepsilon_{\text{total}}}{\varepsilon_{\text{long}}} \implies \varepsilon_{\text{long}} = -V \frac{\varepsilon_{\text{total}}}{V} = \frac{P}{F_0} \]

\[ \frac{r_f - 20 \text{ mm}}{20 \text{ mm}} = -0.000531 \implies r_f = 19.97 \text{ mm} \]

\[ \Delta A = \pi (r_f^2 - r_i^2) = -3.77 \text{ mm}^2 \]
\[ P = 250\text{ kN} \Rightarrow \sigma_{\text{applied}} = \frac{250\text{ kN}}{\pi (20\text{ mm})^2} = 198.54\text{ MPa} \]

Since \( \sigma_{\text{applied}} > 150\text{ MPa} \Rightarrow \text{Plastic Region} \]

\[ \text{"Similar Triangles"} \]
\[ \frac{300 - 150}{0.05 - 0.00125} = \frac{198.54 - 150}{\varepsilon - 0.00125} \Rightarrow \varepsilon' = 0.01716 \]

\[ E' \text{ strain before load is removed} \]
\[ \varepsilon_F \text{ permanent strain, after load is removed} \]

\[ E = 120\text{ GPa} \text{ "Recovery" line} \]

\[ E = \frac{198.54\text{ MPa}}{\varepsilon_F - \varepsilon_F} \Rightarrow \varepsilon_F = 0.0155 \]

\[ \varepsilon_F = \frac{L_F - L_o}{L_o} \Rightarrow L_F = 609.3\text{ mm} \]
Problem 2

4-37. The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load.

\[ \Delta = \Delta_D - \delta_D \]

\[ 0 = \frac{150(10^3)(500)}{4(0.025^2)(200)(10^6)} - \frac{50(10^3)(250)}{4(0.045^2)(101)(10^6)} \]

\[ -\frac{F_D(500)}{4(0.055^2)(101)(10^6)} \quad \frac{F_D(500)}{4(0.025^2)(200)(10^6)} \]

\[ F_D = 107.89 \text{kN} \]

Displacement:

\[ \delta_{A/B} = \frac{P_{AB}L_{AB}}{A_{AB}E_s} = \frac{42.11(10^3)(500)}{4(0.025^2)(200)(10^6)} \]

\[ = 0.335 \text{ mm} \]

Problem 3

5-79. The steel shaft is made from two segments: AC has a diameter of 0.5 in., and CB has a diameter of 1 in. If it is fixed at its ends A and B and subjected to a torque of determine the maximum shear stress in the shaft. \( G_s = 10.8(10^3) \) ksi.

Equilibrium:

\[ T_A + T_B - 500 = 0 \]

(1)

Compatibility condition:

\[ \phi_{D/A} = \phi_{D/B} \]

\[ \frac{T_A(5)}{\frac{5}{2}(0.25^4)G} + \frac{T_A(6)}{\frac{5}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{5}{2}(0.5^4)G} \]

\[ 1408 T_A = 192 T_B \]

(2)

Solving Eqs. (1) and (2) yields

\[ T_A = 60 \text{ lb \cdot ft} \]

\[ T_B = 440 \text{ lb \cdot ft} \]

\[ \tau_{AC} = \frac{T_C}{J} = \frac{50(12)(0.25)}{\frac{5}{2}(0.25^4)} = 29.3 \text{ ksi (max)} \]

\[ \tau_{DB} = \frac{T_C}{J} = \frac{440(12)(0.5)}{\frac{5}{2}(0.5^4)} = 26.9 \text{ ksi} \]

Ans.
Problem 4

Solution:

Step 1: FBD for AB

\[ \sum F_y = 0 \Rightarrow V_A + V_B = 20 \text{ kip}. \]

Symmetric \[ V_A = V_B = 10 \text{ kip}. \]

FBD for BD

\[ \sum M_c = 0 \Rightarrow V_D = -10 \text{ kip}. \]

\[ \sum F_y = 0 \Rightarrow V_c = 30 \text{ kip}. \]
Step 2: \( \bar{y} \) and \( I \)

\[
\bar{y} = \frac{\sum yA}{\sum A}
\]

\[
= \frac{0.25(4x0.5) + 2[2(3x0.5)] + 5.5x10x0.5}{4(0.5) + 2(3x0.5) + 10(0.5)} = 3.40 \text{ in}
\]

\[
I = \frac{1}{12} (4x0.5)^3 + 4(0.5)(3x0.5 - 0.25)^2 + 2\left[\frac{1}{12} 0.5x3^3 + 0.5(3)(3.40 - 2)^2\right] + \frac{1}{12} (0.5x10^3) + 0.5(10x5.5 - 3.40)^2
\]

\[= 91.73 \text{ in}^4\]

Step 3: maximum tensile compressive stress.

\[
\sigma_T = \frac{75x12 - (10.5 - 3.4)}{91.73} = 6.966 \text{ ksi}
\]

\[
\sigma_C = \frac{7.5x12 \times 3.4}{91.73} = 3.336 \text{ ksi}
\]

\[
M_{\sigma_T} = \sigma_T \frac{12.5x12(3.4)}{91.73} = 5.560 \text{ ksi}
\]

\[
\sigma_C = \sigma_C \frac{12.5x12(10.5 - 3.4)}{91.73} = 11.610 \text{ ksi}
\]

Thus: maximum tensile stress at the center of AB, 6.966 ksi.

maximum compressive stress at the point C, 11.610 ksi.
Problem 5

*12-12. Determine the equations of the elastic curve for the beam using the $x_1$ and $x_2$ coordinates. Specify the slope at $A$ and the maximum displacement of the shaft. $EI$ is constant.

Referring to the FBDs of the beam’s cut segments shown in Fig. 2 and 3,

\[ \zeta + \sum M_i = 0; \quad M(x_1) - P x_1 = 0 \quad M(x_2) = P x_2 \]

And

\[ \zeta + \sum M_i = 0; \quad M(x_2) - P a = 0 \quad M(x_2) = P a \]

\[ EI \frac{d^2 u}{dx^2} = M(x) \]

For coordinate $x_1$,

\[ EI \frac{d^3 u_1}{dx_1^3} = P x_1 \]

\[ EI \frac{d u_1}{dx_1} = \frac{P}{2} x_1^2 + C_1 \]

\[ EI u_1 = \frac{P}{6} x_1^3 + C_1 x_1 + C_2 \]  \hspace{1cm} (1)

For coordinate $x_2$,

\[ EI \frac{d^3 u_2}{dx_2^3} = P a \]

\[ EI \frac{d u_2}{dx_2} = P a x_2 + C_3 \]

\[ EI u_2 = \frac{P a}{2} x_2^2 + C_3 x_2 + C_4 \]  \hspace{1cm} (2)

At $x_1 = 0$, $u_1 = 0$. Then, Eq. (2) gives

\[ EI (0) = \frac{P}{6} (0^3) + C_1(0) + C_2 \quad C_2 = 0 \]

Due to symmetry, at $x_2 = \frac{L}{2}$, $\frac{dv_2}{dx_2} = 0$. Then, Eq. (3) gives

\[ EI (0) = Pa \left( \frac{L}{2} \right) + C_3 \quad C_3 = -\frac{PaL}{2} \]

At $x_1 = x_2 = a$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ Thus, Eq. (1) and (3) give

\[ \frac{P}{2} a^2 + C_1 = Pa (a) + \left( \frac{PaL}{2} \right) \]

\[ C_1 = \frac{Pa^2}{2} - \frac{PaL}{2} \]