Vector Addition

To add vectors together they must first be resolved into components. The x (y) component of a vector is found by projecting the vector onto the x (y) axis.

Example: Vector \( \mathbf{A} \) has a length of 5.00 meters and points along the x-axis. Vector \( \mathbf{B} \) has a length of 3.00 meters and points 120° from the +x-axis. Compute \( \mathbf{A} + \mathbf{B} (= \mathbf{C}) \).

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} \\
\cos \theta &= \frac{\text{adj}}{\text{hyp}} \\
\tan \theta &= \frac{\text{opp}}{\text{adj}}
\end{align*}
\]

Example continued

\[
\begin{align*}
\sin 60^\circ &= \frac{B_y}{B} \Rightarrow B_y &= 5.00 \text{ m} \sin 60^\circ = 2.60 \text{ m} \\
\cos 60^\circ &= \frac{B_x}{B} \Rightarrow B_x &= -5.00 \text{ m} \cos 60^\circ = -2.50 \text{ m}
\end{align*}
\]

And \( A_x = 5.00 \text{ m} \) and \( A_y = 0.00 \text{ m} \)

The components of \( \mathbf{C} \):

\[
\begin{align*}
C_x &= A_x + B_x = 5.00 \text{ m} + (-2.50 \text{ m}) = 2.50 \text{ m} \\
C_y &= A_y + B_y = 0.00 \text{ m} + 2.60 \text{ m} = 2.60 \text{ m}
\end{align*}
\]

The length of \( \mathbf{C} \) is:

\[
C = \sqrt{C_x^2 + C_y^2}
\]

From the x-axis
Example: At the instant a traffic light turns green, an automobile starts with a constant acceleration of 2.2 m/s$^2$. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile.

(a) How much time will elapse before the automobile overtakes the truck?

What is the data you are given?
U(truck)=9.5; u(car)=0; $a$(car)=2.2; The distance covered by both are the same; Hence in the same period of time...

(b) How fast will the car be traveling at that instant?

(c) Where do they meet?

Example continued

(b) How fast will the car be traveling at that instant?

(c) Where do they meet?

Example: A penny is dropped from the observation deck of the Empire State Building 369 m above the ground. With what velocity does it strike the ground? Ignore air resistance.

Given: $v_{0y}=0$ m/s; $a_y=-9.8$ m/s$^2$; $y_0=0$ m; and $y_f=-369$ m
Unknown: $v_{yf}$

At t = 3 sec:

<table>
<thead>
<tr>
<th>$v_x$</th>
<th>$v_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.12 m/s</td>
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Example: You throw a ball into the air with speed 15.0 m/s, how high does the ball rise?

Given: $v_{0y}=+15.0$ m/s; $a_y=-9.8$ m/s$^2$

Example: An arrow is shot into the air with $\theta=60^\circ$ (from the horizontal) and $v_0=20.0$ m/s. The arrow is released from a height of 1.80 m above the ground.

(a) What are $v_x$ and $v_y$ of the arrow when $t=3$ sec?

The components of the initial velocity are:

$V_{0x} = V_0 \cos \theta = 10.0$ m/s
$V_{0y} = V_0 \sin \theta = 17.3$ m/s

At $t=3$ sec:

$V_x = V_{0x} = 10.0$ m/s
$V_y = V_{0y} - g t = -12.1$ m/s

(b) What are the $x$ and $y$ components of the displacement of the arrow during the 3.0 sec interval?

$\Delta r=r(\Delta t)-r(t)$
Example continued

The initial position of the arrow is
\[ \mathbf{r}_i = 0 \mathbf{k} + (1.8 \, \text{m}) \mathbf{j} \]

The final position of the arrow is
\[ \begin{align*}
  x_f &= x_i + v_{x_0}t = 30.0 \, \text{m} \\
  y_f &= y_i + v_{y_0}t - \frac{1}{2} gt^2 = 9.60 \, \text{m}
\end{align*} \]

The displacement is
\[ \Delta \mathbf{r} = (30 \, \text{m}) \mathbf{k} + (7.8 \, \text{m}) \mathbf{j} \]

Example: How high does the arrow go?

The arrow rises until the y-coordinate of the arrow is
\[ y(t = 1.77 \, \text{s}) = y_i + v_{y_0}t - \frac{1}{2} gt^2 = 17.1 \, \text{m} \]

Relative motion

Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?

From the picture:
\[ \Delta \mathbf{r}_{AB} = \Delta \mathbf{r}_{AG} + \Delta \mathbf{r}_{BG} \]

\[ v_{BA} = v_{AG} - v_{BG} \]
\[ v_{BA} = 65 \, \text{miles/hr east} - 60 \, \text{miles/hr east} = 5 \, \text{miles/hr east} \]

Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?

From the picture:
\[ \Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{AB} + \Delta \mathbf{r}_{BA} \]

\[ v_{BA} = v_{AB} - v_{BA} \]
\[ v_{BA} = 65 \, \text{miles/hr east} + 60 \, \text{miles/hr west} = 125 \, \text{miles/hr east} \]
Example continued:

From the picture:

$$\Delta r_{AG} = \Delta r_{BG} = \Delta r_{BA}$$

Divide by $\Delta t$: 

$$\frac{\Delta r_{AG}}{\Delta t} = \frac{\Delta r_{BG}}{\Delta t} = \frac{\Delta r_{BA}}{\Delta t}$$

$$65 \text{ miles/hr west} = 60 \text{ miles/hr east}$$

$$125 \text{ miles/hr west}$$

From the picture:

$$\Delta r_{AG} = \Delta r_{BG} = \Delta r_{BA}$$

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$$\frac{\Delta r_{AG}}{\Delta t} = \frac{\Delta r_{BG}}{\Delta t} = \frac{\Delta r_{BA}}{\Delta t}$$

$$65 \text{ miles/hr west} - 60 \text{ miles/hr east}$$

$$125 \text{ miles/hr west}$$

Example continued:

In still water: 

$$t = \frac{d}{v} = \frac{2000 \text{ m}}{1.2 \text{ m/s}} = 1670 \text{ s}$$

Or 435 seconds per leg

Example continued:

Example: A jet moving initially with $v = 300 \text{ mph}$ due east enters a region where the wind is blowing at $100 \text{ mph}$ at $30^\circ$ north of east. What is the new velocity of the jet?

Place the vectors tip-to-tail:

The magnitude and direction of the velocity are

$$v = 390 \text{ mph, 7.4° north of east}$$

Example continued:

$$v_{\text{wind}} = v_{\text{jet}} + v_{\text{wind}}$$

$$= (300 \text{ mph}) \hat{k} + (100 \text{ mph}) \cos 30^\circ \hat{k} + (100 \text{ mph}) \sin 30^\circ \hat{j}$$

$$= (387 \text{ mph}) \hat{k} + (50 \text{ mph}) \hat{j}$$

The magnitude and direction of the velocity are

$$|v| = \sqrt{v_x^2 + v_y^2} = 390 \text{ mph}$$

$$\tan \theta = \frac{v_y}{v_x} = 0.129$$

The plane travels $390 \text{ mph, 7.4° north of east}$

Free Body Diagrams

Use idealized models to account for all forces acting on each mass (body) involved in the system being analyzed.
Example: Find the tension in each cord of the system shown in the figure.

![Image of a system with tensions in cords]

Example: A box slides across a rough surface. If the coefficient of kinetic friction is 0.3, what is the acceleration of the box?

\[ \sum F = N_a + f_{sk} + w_a = ma \]

Apply Newton's 2nd Law:

\[ \sum F = -f_{sk} = ma \]

\[ \sum F = N_a - w_a = 0 \]

Example continued

1. \[-f_{sk} = ma \]
2. \[N_a - w_a = 0 \]

From (1):

\[-f_{sk} = -\mu_s N_a = -\mu_s mg = ma \]

Solving for \(a\):

\[a = -\mu_s g \]

\[= -(0.3)(9.8 \text{ m/s}^2) = -2.94 \text{ m/s}^2\]

Example: A 1.00 kg mass is at rest on a ramp that makes an angle of 20° with respect to the horizontal. For this situation, \(\mu_s = 0.400\). What is the magnitude of the static friction force?

\[ N \cos \theta = mg \]

\[ N \sin \theta = f_s \]

From (1):

\[N \cos \theta = mg \]

\[N = \frac{mg}{\cos \theta} \]

\[f_s = \mu_s N \]

\[f_s = \mu_s \frac{mg}{\cos \theta} \]

\[f_s = \mu_s mg \tan \theta \]

Example continued

1. \[\sum F = -f_{sk} + mg \sin \theta = 0 \]
2. \[\sum F = N_a - mg \cos \theta = 0 \]

The magnitude of the static friction force can be found from (1):

\[f_{sk} = mg \sin \theta = 3.35 \text{ N}\]

What is the magnitude of the maximum static friction force?

\[f_{sk} = \mu_s mg \cos \theta = 3.68 \text{ N}\]
Example continued

If the angle of the ramp is changed to 40°, what is the acceleration of the mass? Take $\mu_k = 0.35$.

The FBD is unchanged, except $f_{rb}$ is now the kinetic friction force of the ramp on the box.

Apply Newton's 2nd law $\sum F = ma$

(1) $\sum F_x = -f_{rb} + mg \sin \theta = ma$

(2) $\sum F_y = N_k - mg \cos \theta = 0$

Example continued

b. Apply Newton's Second law to the two masses.

Example: A 3.00 kg mass rests on a frictionless tabletop. This mass is connected to a 5.00 kg mass by a light string as shown. Assume the pulley is massless.

a. Draw free body diagrams for the two masses and the pulley.

Dynamics of Circular Motion

$r(t) = r \cos \theta \hat{x} + r \sin \theta \hat{y}$

$v(t) = -r \sin \theta \hat{x} + r \cos \theta \hat{y}$

$a(t) = -r \omega^2 \cos \theta \hat{x} - r \omega^2 \sin \theta \hat{y}$

Rotational motion can be related to translational motion by converting between Cartesian and polar coordinate systems.

Example: The Hubble Space Telescope orbits the Earth at an altitude of about 600 km with an orbit period of about 100 minutes, what is Hubble's orbital speed? (Assume a circular orbit.)

The tangential velocity and radial acceleration of a body (constant speed case).

$\nu = \frac{\text{total distance}}{\text{total time}} = \frac{rA}{\Delta t} = \frac{2\pi}{T} = 7300 \text{m/s}$

$r = R_e + h = 6.38 \times 10^6 \text{m}$

$T = 6000 \text{ s}$
Example continued

(b) What is HST’s angular speed?

\[
\omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{1} = 1.05 \times 10^3 \text{ rad/sec}
\]

Example: What is the magnitude of the radial acceleration of HST?

\[
\vec{a}_r = \frac{\vec{V}^2}{r} = 7.63 \text{ m/s}^2 = 0.78 \text{ g}
\]

Previously: Consider an object in uniform circular motion (speed=constant).

\[
\begin{align*}
\vec{r}(t) &= r \cos \theta \hat{x} + r \sin \theta \hat{y} \\
\vec{v}(t) &= -r \omega \sin \theta \hat{x} + r \omega \cos \theta \hat{y} \\
\vec{a}(t) &= -r \omega^2 \cos \theta \hat{x} - r \omega^2 \sin \theta \hat{y}
\end{align*}
\]

The magnitude of the (tangential) velocity is \( \nu = r \omega \).

The magnitude of the (radial) acceleration is \( a_r = r \omega^2 = \nu^2 / r \).

Example continued

(b) If \( \mu_s = 0.40 \) and the cylinder has \( r = 2.5 \text{ m} \), what is the minimum angular speed of the cylinder so that the people don't fall out?

Apply Newton's 2nd law:

\[
\sum F = \sum m \vec{a}
\]

(1) \( \sum F_c = N - mg = ma \)

(2) \( \sum F_r = f_{cw} - w_c = 0 \)

From (2):

\[
\sum f_{cw} = w_c = mg \quad \text{from (1)}
\]

\[
\mu_s N_{cw} = \mu_s (ma \cdot r) = mg
\]

[\[
\omega = \frac{\sqrt{\frac{9.8 \text{ m/s}^2}{0.40(2.5 \text{ m})}}}{3.13 \text{ rad/s}}
\]

Example: The rotor is an amusement park ride where people stand against the inside of a cylinder. Once the cylinder is spinning fast enough the floor drops out.

(a) What force keeps the people from falling out the bottom of the cylinder?

It is the force of static friction.

Example: A coin is placed on a record that is rotating at 33.3 rpm. If \( \mu_k = 0.1 \), how far from the center of the record can the coin be placed without having it slip off?

Apply Newton's 2nd law:

\[
\sum F = \sum m \vec{a}
\]

(1) \( \sum F_c = N - mg = ma \)

(2) \( \sum F_r = f_{cw} - w_c = 0 \)
Example continued

From (1): \( f_{cc} = \mu N_c = \mu (mg) = ma^2r \)

Solving for \( a \):
\[ a = \frac{\mu g}{\omega^2} \]

What is \( \omega \)?

\[ \omega = \frac{2 \pi \text{ rad}}{1 \text{ min}} = \frac{3.5 \text{ rad/s}}{1 \text{ rev}} \]

\[ a = \frac{0.1(9.8 \text{ m/s}^2)}{3.5 \text{ rad}^2/\text{s}^2} = 0.08 \text{ m} \]

Example continued

The apparent weight at the top of loop is:
\[ N_c + mg = \frac{v^2}{r} \]

\[ N_c = \frac{v^2}{r} - g \]

\[ N_c = 0 \text{ when } \frac{v^2}{r} = g \]

This is the minimum speed needed to make it around the loop.

Example continued

From (2) the normal force is:
\[ N_c = mg \cos \theta \]

Using (1):
\[ -f_{cb} + mg \sin \theta = ma \]

\[ -\mu N_c + mg \sin \theta = ma \]

\[ -\mu (mg \cos \theta) + mg \sin \theta = ma \]

\[ a = g(\sin \theta - \mu_c \cos \theta) = 3.67 \text{ m/s}^2 \]
Work and Energy

Example: The extinction of the dinosaurs and the majority of species on Earth in the Cretaceous Period (65 Myr ago) is thought to have been caused by an asteroid striking the Earth near the Yucatan Peninsula. The resulting ejecta caused widespread global climate change.

If the mass of the asteroid was $10^{16}$ kg (diameter in the range of 4-9 miles) and had a speed of 30.0 km/sec, what was the asteroid's kinetic energy?

$$ K = \frac{1}{2} m v^2 = \frac{1}{2} (10^{16} \text{ kg})(30 \times 10^3 \text{ m/s})^2 $$

$$ = 4.5 \times 10^{22} \text{ J} $$

This is equivalent to ~10^9 Megatons of TNT.

Example: What is the net work done on a box of mass $m$ that is being pushed along a rough surface as shown?

The work done by the pushing force is:

$$ W_p = F \cdot \Delta x = (F \cos \theta) \Delta x $$

The work done by the Normal force is:

$$ W_N = N \cdot \Delta x = 0 $$

The normal force is perpendicular to the displacement.

The work done by gravity is:

$$ W_g = w \cdot \Delta x = 0 $$

The force of gravity is perpendicular to the displacement.

The work done by kinetic friction is:

$$ W_{fr} = F_{fr} \cdot \Delta x = -\mu_k (mg + F \sin \theta) \Delta x $$

Example continued

The forces and displacement (in unit vector notation) are

$$ F = F \cos \theta \hat{x} - F \sin \theta \hat{y} $$

$$ w = w \hat{y} $$

$$ N = N \hat{y} $$

$$ f_{fr} = -\mu_k (mg + F \sin \theta) \hat{x} $$

$$ \Delta x = \Delta t \hat{x} $$

The work done by the pushing force is:

$$ W_p = F \cdot \Delta x = (F \cos \theta) \Delta x $$

The net work done on the box is:

$$ W_{net} = W_p + W_N + W_g + W_{fr} $$

$$ = (F \cos \theta) \Delta x + 0 - \mu_k (mg + F \sin \theta) \Delta x $$

$$ = (F \cos \theta - \mu_k mg - \mu_k F \sin \theta) \Delta x $$
Example: A ball is tossed straight up. What is the work done by the force of gravity on the ball as it rises?

\[
W_g = w_a \cdot \Delta r = w_a \Delta y \cos 180^\circ = -mg\Delta y
\]

\(W < 0\) and the KE of the ball decreases.

Example: An ideal spring has \(k = 20.0 \text{ N/m}\). What is the amount of work done (by an external agent) to stretch the spring 0.40 m from its relaxed length?

\[
W = \int F \cdot dr = \int F dx = \frac{1}{2} kx^2 = \frac{1}{2} (20.0 \text{ N/m})(0.40 \text{ m})^2 = 1.6 \text{ J}
\]

Example: A 4.00 kg particle moves along the \(x\)-axis. Its position varies with time according to \(x(t) = t + 2t^3\), where \(x\) is measured in meters and \(t\) is in seconds.

(a) What is the particle's kinetic energy?

(b) What is the particle's acceleration? What is the net force acting on the particle?

(c) The power delivered to the particle at time \(t = 0\)?

(d) The work done on the particle from \(t = 0\) to \(t = 2\) sec?

Example: A box of mass \(m\) is towed up a frictionless incline at constant speed. The applied force \(F\) is parallel to the incline. What is the net work done on the box?

Apply Newton's 2\(^{nd}\) law:

\[
\sum F_x = F_{\text{net}} = -w_a \sin \theta = 0
\]

\[
\sum F_y = N_a - w_a \cos \theta = 0
\]