1.) Find a general solution the following differential equation

\[ y'' + 3y' = 5x^2 e^{-3x} + x\sin(x) \]

2.) Solve the following differential equation

\[ y'' - 6y' + 9y = e^{3x} \quad y(0) = 0 \quad y'(0) = 1 \]

3.) We have a tank that initially contains 400 gallons of water and 20 pounds of salt. Pure water enters the tank at a rate of 5 gal/min, a well stirred mixture leaves the tank at a rate of 7 gal/min. Set up an initial value problem and solve the IVP.

4.) We have a 7th order homogeneous differential equation with roots \( r = 0 \) with multiplicity 3, \( r = -2 \) with multiplicity 2 and complex conjugates \( r = 3 \pm 2i \). Find a general solution.

5.) Solve the following IVP problems:

\[ \frac{y'}{1 + 2x^2} = x \quad y(1) = -2 \]

\[ x^2 y' = (x - 3)y^2 \quad y(1) = 3 \]

\[ (1 + x^2)y' + xy = x \quad y(0) = 0 \]

6.) Newton’s Law of Heating says that the rate of change of temperature of an object is proportional the difference between the temperature of the object and the temperature of the surrounding air. A roast is taken from the refrigerator which as a temperature of 40 degrees Fahrenheit and put it in an oven that is 350 degrees Fahrenheit.

   a.) Write a differential Equation that describes the rate of change of the temperature
   b.) After one hour the meat thermometer shows a temperature of 90 degrees Fahrenheit. Use this information and part a. to find solution.
   c.) If the roast is done when it is 140 degrees Fahrenheit, how long should the roast be left in the oven?
   d.) What is the equilibrium solution for this differential equation?

7.) Solve the following homogeneous differential equations:

\[ y'' - 4y = 0 \quad y(0) = 3 \quad y'(0) = 0 \]
\[ y'' + 6y' + 9y = 0 \quad y(0) = 1 \quad y'(0) = -4 \]
\[ y'' - 6y' + 13y = 0 \quad y(0) = 2 \quad y'(0) = 1 \]

8.) Find the General Solution to the following Bernoulli Differential Equation:

\[ 2yy' + y^2 = -\left(x^2 + x\right) \]

9.) Apply Euler’s method to approximate the solution to \( y(1) \) on the interval \([0,1]\) with \( h=0.5 \). Compare the error at each step to the given exact solution:

\[ y' = 2xy \quad y(0) = 1 \]
1.) \( y = A \cos(2x) + B \sin(2x) + x (C \cos(2x) + D \sin(2x)) + e^{-3x} + F e^{-3x} + G x^3 e^{-3x} + c_1 + c_2 e^{-3x} \)

2.) \( y = x e^{3x} + \frac{1}{2} x^2 e^{3x} \)

3.) \( \frac{ds}{dt} = -\frac{7s}{400 - 2t}, \quad s(t) = \sqrt{400 - 2t} \)

4.) \( y = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x} + c_5 x e^{-2x} + e^{-3x} (c_6 \cos(2x) + c_7 \sin(2x)) \)

5.)

(1) \( y = -\sqrt{x^4 + x^2 + 2} \)

(2) \( y = -\frac{3x}{-10x + 3 \ln(x) + 9} \)

(3) \( y = 1 - \frac{1}{\sqrt{1 + x^2}} \)

6.)

a.) \( \frac{dT}{dt} = k(350 - T) \)

b.) \( T = 350 - 310 e^{\ln(\frac{26}{31})} t \)

c.) \( t \approx 2.21 \) s

d.) \( T = 350 \)

7.)

(1) \( y = \frac{3}{2} e^{2x} + \frac{3}{2} e^{-2x} \)

(2) \( y = 7xe^{-3x} + e^{-3x} \)

(3) \( y = \frac{\sin(\sqrt{7}x)}{\sqrt{7}} + 2 \cos(\sqrt{7}x) \)

8.) \( y = \sqrt{-\left(1 - x + x^2\right) + Ce^{-x}} \)

9.)

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